The synthesis of Single-Subject Experimental Data: Extensions of the Basic Multilevel Model

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Outline

1. Analysis of data from one subject
2. Multilevel analysis of SSED data
3. Study 1: autocorrelation
4. Study 2: nonlinear trajectories
5. Study 3: heterogeneity at level 1
6. Study 4: covariance misspecification
7. Conclusions

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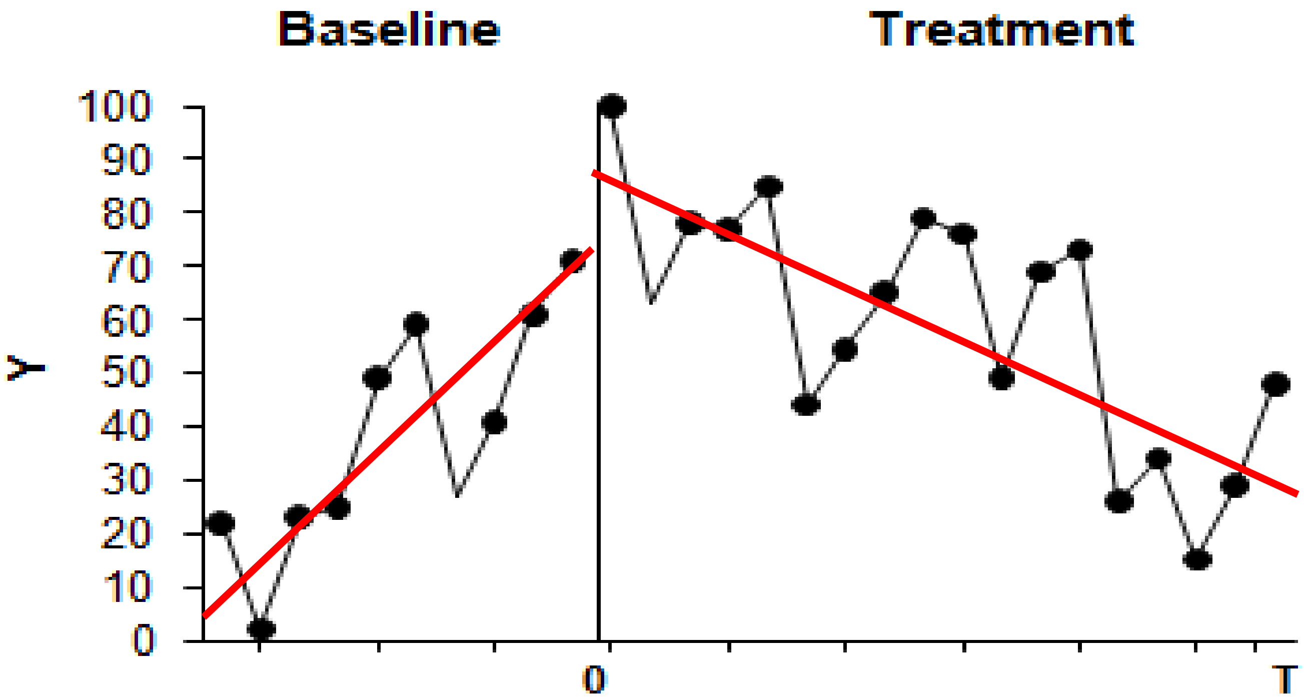
1. Analysis of data from one subject

$$\mathbf{Y_i = \beta_0 + \beta_1 Time_i + \beta_2 Treatment_i + \beta_3 (Time_i * Treatment_i) + e_i}$$

(Center, Skiba & Casey, 1985-1986)



$$\hat{Y}_i = \beta_0 + \beta_1 Time_i \qquad \hat{Y}_i = (\beta_0 + \beta_2) + (\beta_1 + \beta_3) Time_i$$

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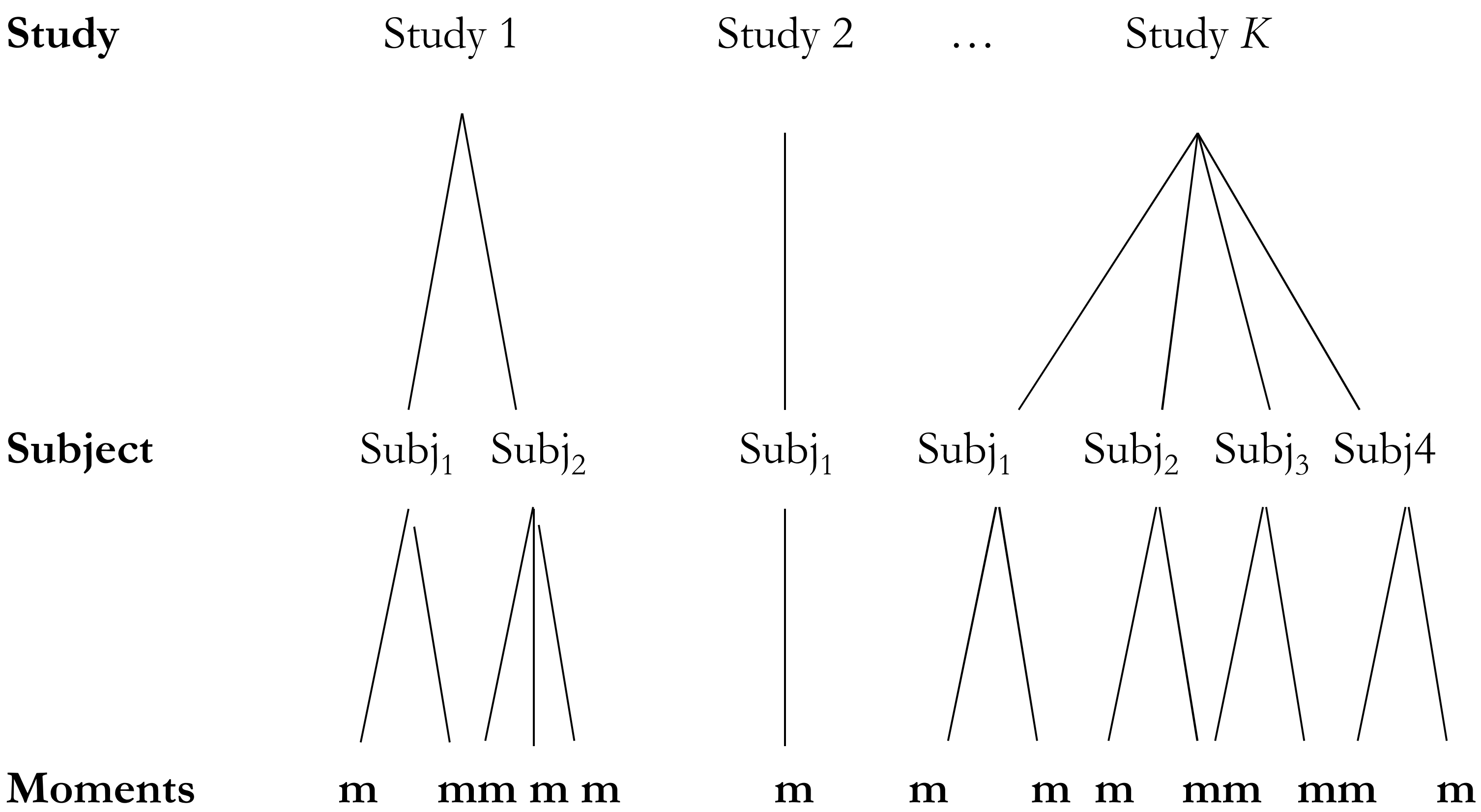
2. Multilevel analysis of SSED data



2. Multilevel analysis of SSED data

$$Y_{ijk} = \beta_{0jk} + \beta_{1jk} Time_{ijk} + \beta_{2jk} Treatment_{ijk} + \beta_{3jk} Time_{ijk} Treatment_{ijk} + e_{ijk}$$

$$e_{ijk} \sim N(0, \sigma_e^2)$$

$$\begin{cases} \beta_{0jk} = \theta_{0k} + u_{0jk} \\ \beta_{1jk} = \theta_{1k} + u_{1jk} \\ \beta_{2jk} = \theta_{2k} + u_{2jk} \\ \beta_{3jk} = \theta_{3k} + u_{3jk} \end{cases} \qquad \begin{bmatrix} u_{0jk} \\ u_{1jk} \\ u_{2jk} \\ u_{3jk} \end{bmatrix} \sim N(0, \Sigma_u)$$

$$\begin{cases} \theta_{0k} = \gamma_0 + v_{0k} \\ \theta_{1k} = \gamma_1 + v_{1k} \\ \theta_{2k} = \gamma_2 + v_{2k} \\ \theta_{3k} = \gamma_3 + v_{3k} \end{cases} \qquad \begin{bmatrix} v_{0k} \\ v_{1k} \\ v_{2k} \\ v_{3k} \end{bmatrix} \sim N(0, \Sigma_v)$$

2. Multilevel analysis of standardized SSED data

$$Y_{ijk} = \beta_{0jk} + \beta_{1jk} Time_{ijk} + \beta_{2jk} Treatment_{ijk} + \beta_{3jk} Time_{ijk} Treatment_{ijk} + e_{ijk}$$

$$e_{ijk} \sim N(0, \sigma^2_{e_{jk}})$$

$$Y^*_{ijk} = \frac{Y_{ijk}}{\sigma_{e_{jk}}} = \beta^*_{0jk} + \beta^*_{1jk} Time_{ijk} + \beta^*_{2jk} Treatment_{ijk} + \beta^*_{3jk} Time_{ijk} Treatment_{ijk} + e^*_{ijk}$$

$$e^*_{ijk} \sim N(0, 1)$$

Simulation results:

- Multilevel approach performs well, even for small *K*, *J* and *I*
- Simple standardization not recommended if *I* < 30
- Power depends largely on
  - Between study variance
  - Number of studies (preferably at least 30)

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Model

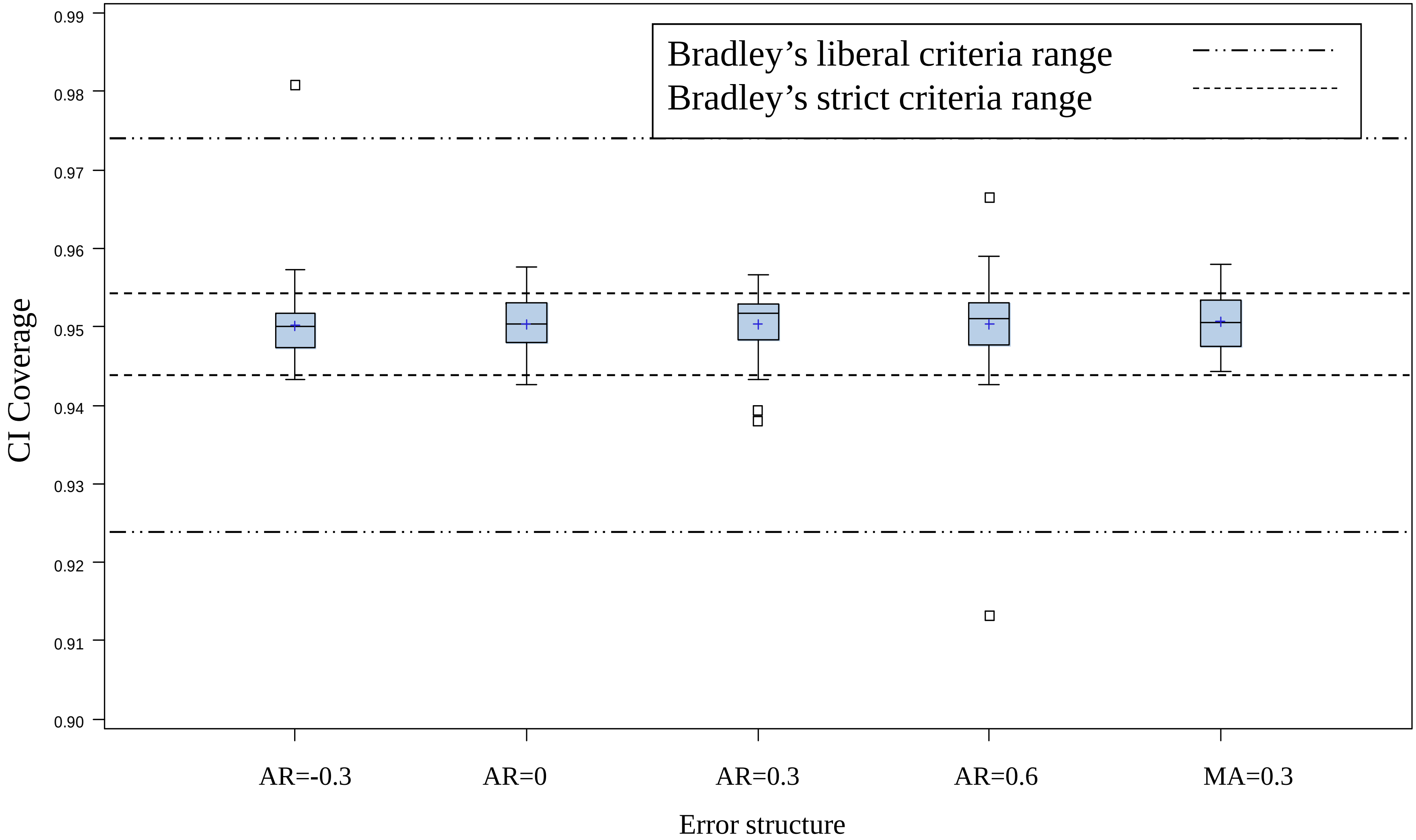
$$Y_{ijk} = \beta_{0jk} + \beta_{1jk} Time_{ijk} + \beta_{2jk} Treatment_{ijk} + \beta_{3jk} Time_{ijk} Treatment_{ijk} + e_{ijk}$$

$$e_{ijk} \overset{iid}{\sim} N(0, \sigma_e^2)$$

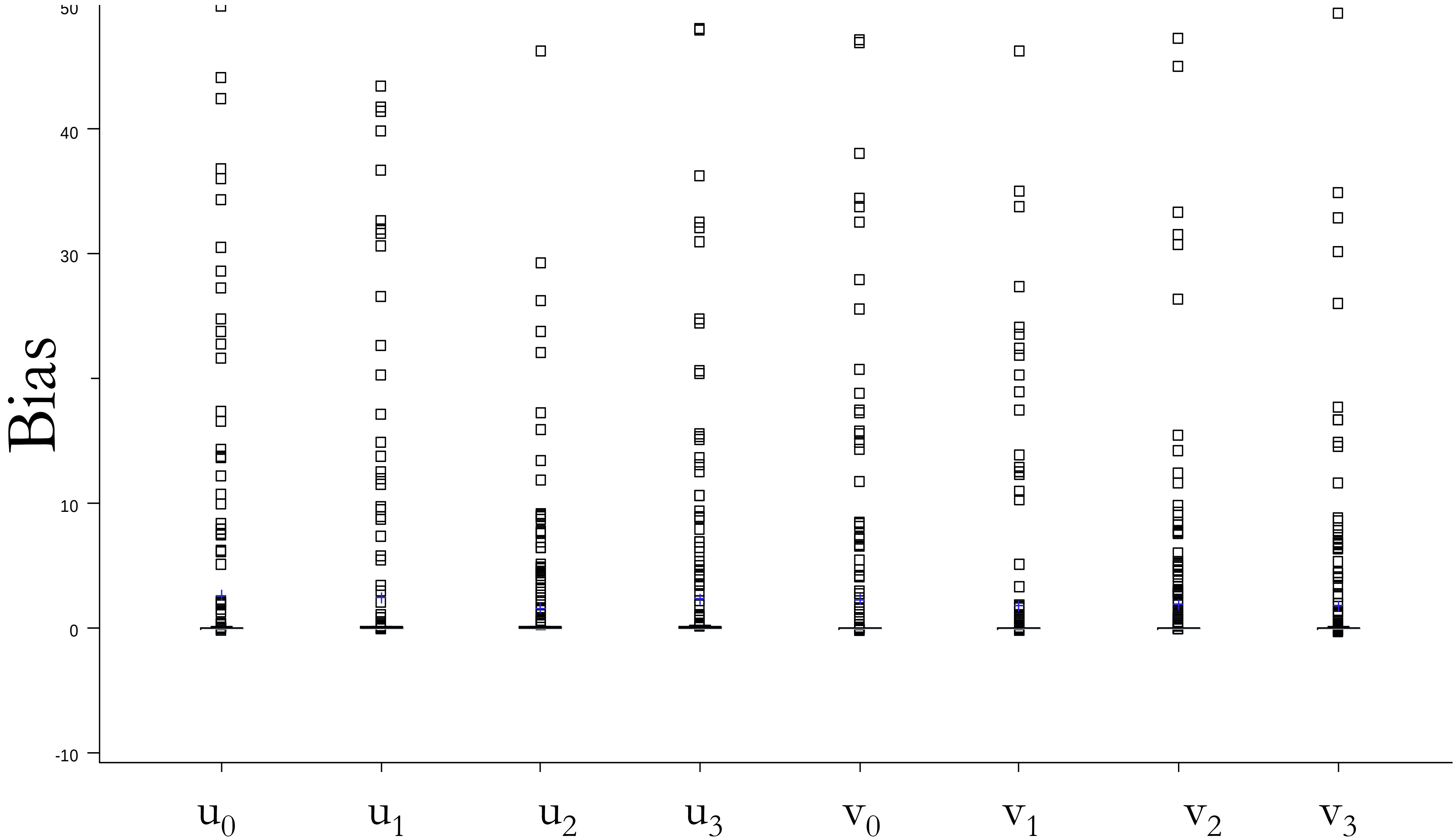
$$\begin{cases} \beta_{0jk} = \theta_{0k} + u_{0jk} \\ \beta_{1jk} = \theta_{1k} + u_{1jk} \\ \beta_{2jk} = \theta_{2k} + u_{2jk} \\ \beta_{3jk} = \theta_{3k} + u_{3jk} \end{cases} \qquad \begin{bmatrix} u_{0jk} \\ u_{1jk} \\ u_{2jk} \\ u_{3jk} \end{bmatrix} \sim N(0, \Sigma_u)$$

$$\begin{cases} \theta_{0k} = \gamma_0 + v_{0k} \\ \theta_{1k} = \gamma_1 + v_{1k} \\ \theta_{2k} = \gamma_2 + v_{2k} \\ \theta_{3k} = \gamma_3 + v_{3k} \end{cases} \qquad \begin{bmatrix} v_{0k} \\ v_{1k} \\ v_{2k} \\ v_{3k} \end{bmatrix} \sim N(0, \Sigma_v)$$

Simulation study
(confidence interval coverage)



Simulation study



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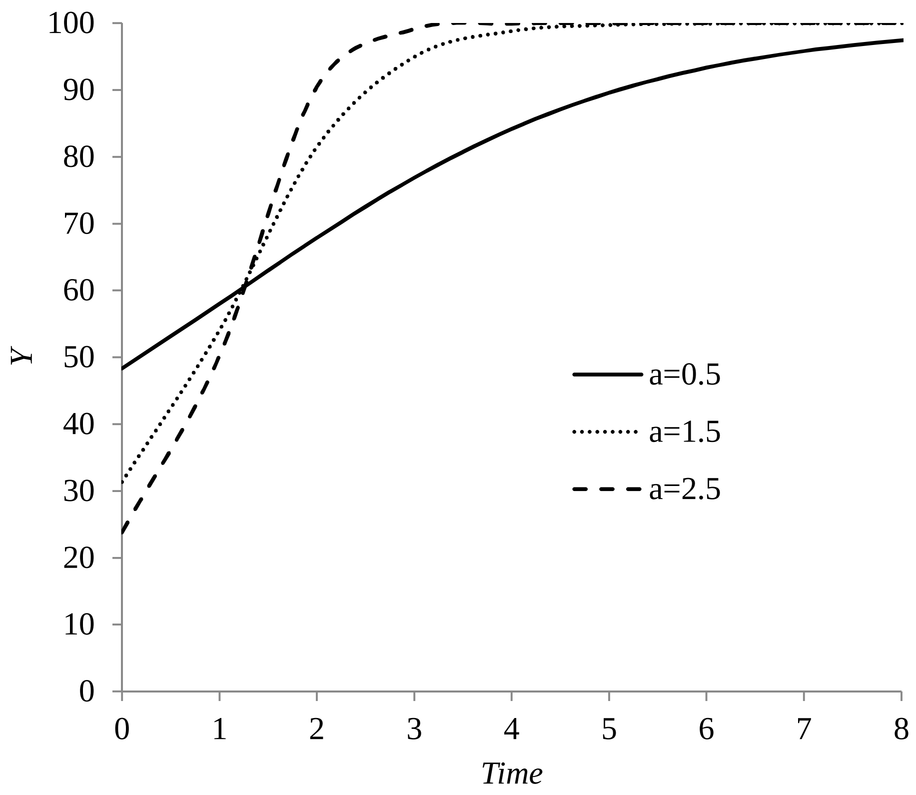
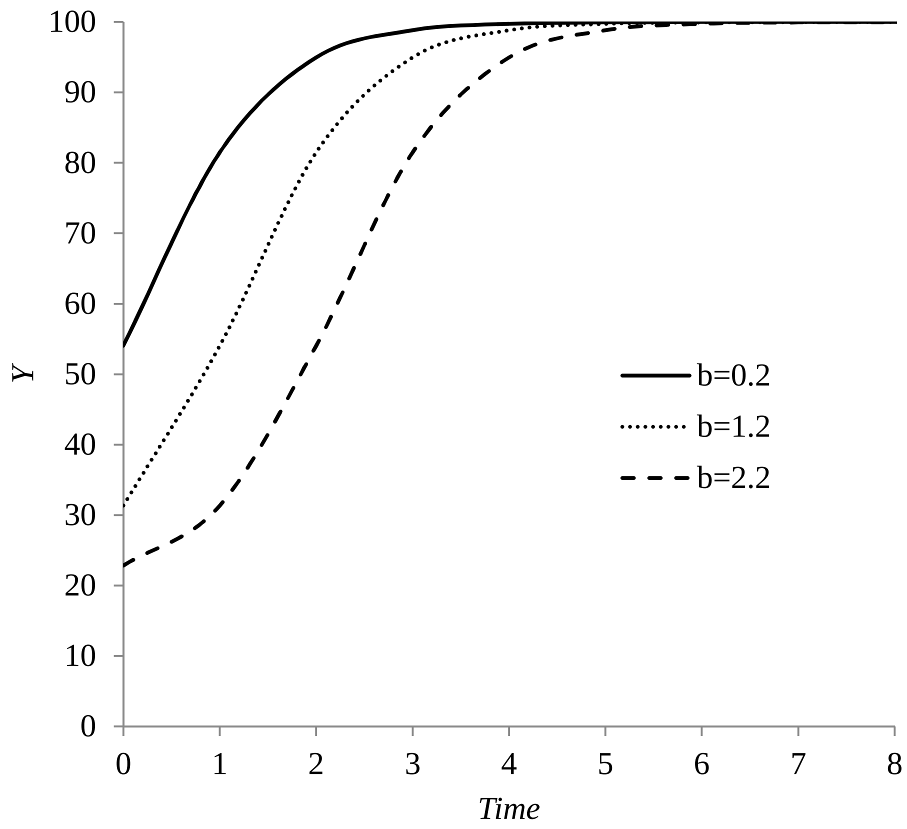
Model

$$Y_{ti} = \gamma_{00} + u_{0i} + (1 - D_{ti})(e_{A,ti}) + (D_{ti})\left(\frac{[f - (\gamma_{00} + u_{0i})]}{\{1 + \exp[-a(Time_{ti} - b)]\}} + e_{B,ti}\right)$$





Simulation study

- Data generation: Two-level logistic model
- Data analysis (Bayesian):
  - Logistic
  - Quadratic model

$$Y_{ti} = \gamma_{00} + u_{0i} + (1 - D_{ti})(e_{A,ti})$$
$$+ (D_{ti})(\gamma_{10,Q} + \gamma_{20,Q} Time_{ti} + \gamma_{30,Q} Time_{ti}^2 + e_{B,ti})$$

  - 'Level change' model:

$$Y_{ti} = \gamma_{00} + u_{0i} + (1 - D_{ti})(e_{A,ti})$$
$$+ (D_{ti})(\gamma_{10,\Delta L} + e_{B,ti})$$

Results

Model with lowest DIC value

J	I	Log-Uni	Log-HC	ΔLevels	Quadratic
3	10	9.6%	7.8%	0.7%	**81.9%**
	20	11.4%	9.6%	0.8%	**78.2%**
	40	10.2%	7.3%	4.5%	**78.0%**
4	10	4.0%	3.7%	0.3%	**92.0%**
	20	8.0%	5.4%	0.2%	**86.4%**
	40	4.4%	3.8%	2.3%	**89.5%**
7	10	1.1%	0.8%	0.0%	**98.1%**
	20	1.5%	1.1%	0.0%	**97.4%**
	40	1.9%	0.7%	0.0%	**97.4%**

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Model
(heterogeneity over phases and 2 settings)

$$
\begin{aligned}
Y_{ijk} = \beta_{0jk} &+ (e_{A,ijk})(1 - D_{ijk}) \\
&+ (\beta_{1jk} + e_{BS1,ijk})(S1_i)(D_{ijk}) \\
&+ (\beta_{2jk} + e_{BS2,ijk})(S2_i)(D_{ijk})
\end{aligned}
$$

Simulation study

- Three-level data generated & standardized using RMSE assuming :
  - homoscedasticity
  - heteroscedasticty over phases
  - Heteroscedasticity over phases & settings
- Hedges' (1981) bias correction

$$Y_j^{(S)} = \left[\frac{Y_j}{RMSE_j} \right] \left[1 - \frac{3}{4(m_j) - 1} \right]$$

- Evaluation of parameter and SE bias.

Results

- If homoscedastic: all models provided unbiased estimates of fixed effects and associated SEs.

- If heteroscedastic: modeling heteroscedasticity yields unbiased estimates of fixed effects and SEs

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Simulation study

		Model Used to Generate Data	
		Without covariance	With covariance
Model to analyze the data	Without covariance	**Scenario A1**	**Scenario B1**
	With covariance	**Scenario A2**	**Scenario B2**

Results (standard errors)

			Relative Standard Error Biases			
			$J=4$		$J=7$	
Scenario	K		$\sigma^2_{v_2}=2$	$\sigma^2_{v_2}=8$	$\sigma^2_{v_2}=2$	$\sigma^2_{v_2}=8$
A1	10	$\sigma^2_{u_2}=2$	-0.05	-0.03	-0.06	-0.02
		$\sigma^2_{u_2}=8$	-0.04	-0.01	0.02	0.00
	30	$\sigma^2_{u_2}=2$	-0.03	0.06	-0.07	-0.02
		$\sigma^2_{u_2}=8$	-0.04	-0.03	-0.06	0.02
A2	10	$\sigma^2_{u_2}=2$	-0.04	-0.04	-0.01	-0.08
		$\sigma^2_{u_2}=8$	-0.06	-0.03	-0.06	-0.08
	30	$\sigma^2_{u_2}=2$	-0.05	-0.03	0.01	-0.01
		$\sigma^2_{u_2}=8$	-0.05	-0.01	-0.05	-0.04
B1	10	$\sigma^2_{u_2}=2$	**-0.17**	**-0.18**	**-0.13**	**-0.15**
		$\sigma^2_{u_2}=8$	**-0.22**	**-0.19**	**-0.18**	**-0.19**
	30	$\sigma^2_{u_2}=2$	**-0.20**	**-0.12**	**-0.15**	**-0.10**
		$\sigma^2_{u_2}=8$	**-0.19**	**-0.23**	**-0.14**	-0.09
B2	10	$\sigma^2_{u_2}=2$	**-0.22**	-0.09	**-0.10**	**-0.12**
		$\sigma^2_{u_2}=8$	**-0.22**	**-0.23**	**-0.13**	**-0.13**
	30	$\sigma^2_{u_2}=2$	**-0.22**	**-0.14**	-0.08	-0.06
		$\sigma^2_{u_2}=8$	**-0.24**	**-0.11**	**-0.15**	**-0.09**

Results (variance estimates)

			$\hat{\sigma}^2_{u_2}$			
			$K = 10$		$K = 30$	
Scenario	J	$\sigma^2_{u_2}$	$\sigma^2_{v_2} = 2$	$\sigma^2_{v_2} = 8$	$\sigma^2_{v_2} = 2$	$\sigma^2_{v_2} = 8$
A1	4	2	-0.05	-0.08	-0.03	-0.02
		8	-0.04	-0.02	0.00	0.00
	7	2	-0.03	-0.05	-0.01	0.00
		8	0.01	0.00	-0.01	0.00
A2	4	2	-0.06	-0.08	0.01	-0.04
		8	-0.04	-0.03	-0.02	-0.01
	7	2	-0.01	0.02	-0.02	-0.01
		8	-0.01	-0.01	0.00	-0.01
B1	4	2	**-0.47**	**-0.50**	**-0.44**	**-0.43**
		8	**-0.18**	**-0.18**	**-0.16**	**-0.13**
	7	2	**-0.46**	**-0.45**	**-0.45**	**-0.43**
		8	**-0.18**	**-0.15**	**-0.15**	**-0.15**
B2	4	2	-0.06	-0.03	-0.02	-0.02
		8	-0.06	-0.03	-0.01	0.00
	7	2	-0.03	-0.02	-0.01	0.00
		8	-0.01	-0.01	-0.01	-0.01

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- High flexibility of multilevel framework
- Good performance if correctly specified, even for relative small samples
- Results are relatively robust (esp. for fixed effects)
- Open question: how complex can we make the model?

References

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Thank you!